**What is a quadratic equation?**

A quadratic equation is defined as an equation where the highest power of the unknown amount (eg x) is 2.

$$ax^2 + bx + c = 0$$

This is the general formula for a quadratic equation provided that **$a \neq 0$.**

The following are forms of quadratic equations:

$x^2 + 7 = 0$

$(x + 7)^2 = 0$

$3x^2 = 7$

$5x^2 = 8x - 76$

$3x^2 + 8x = 12$

$7x = 87 - 2x^2$

If the highest power is more than two, then the equation is not quadratic.

Quadratic equations are one of a set of polynomial equations which include straight line graphs, quadratic and cubic equations. There are lots more of these. A polynomial equation is an equation that can be expressed in positive integer exponents (or powers).

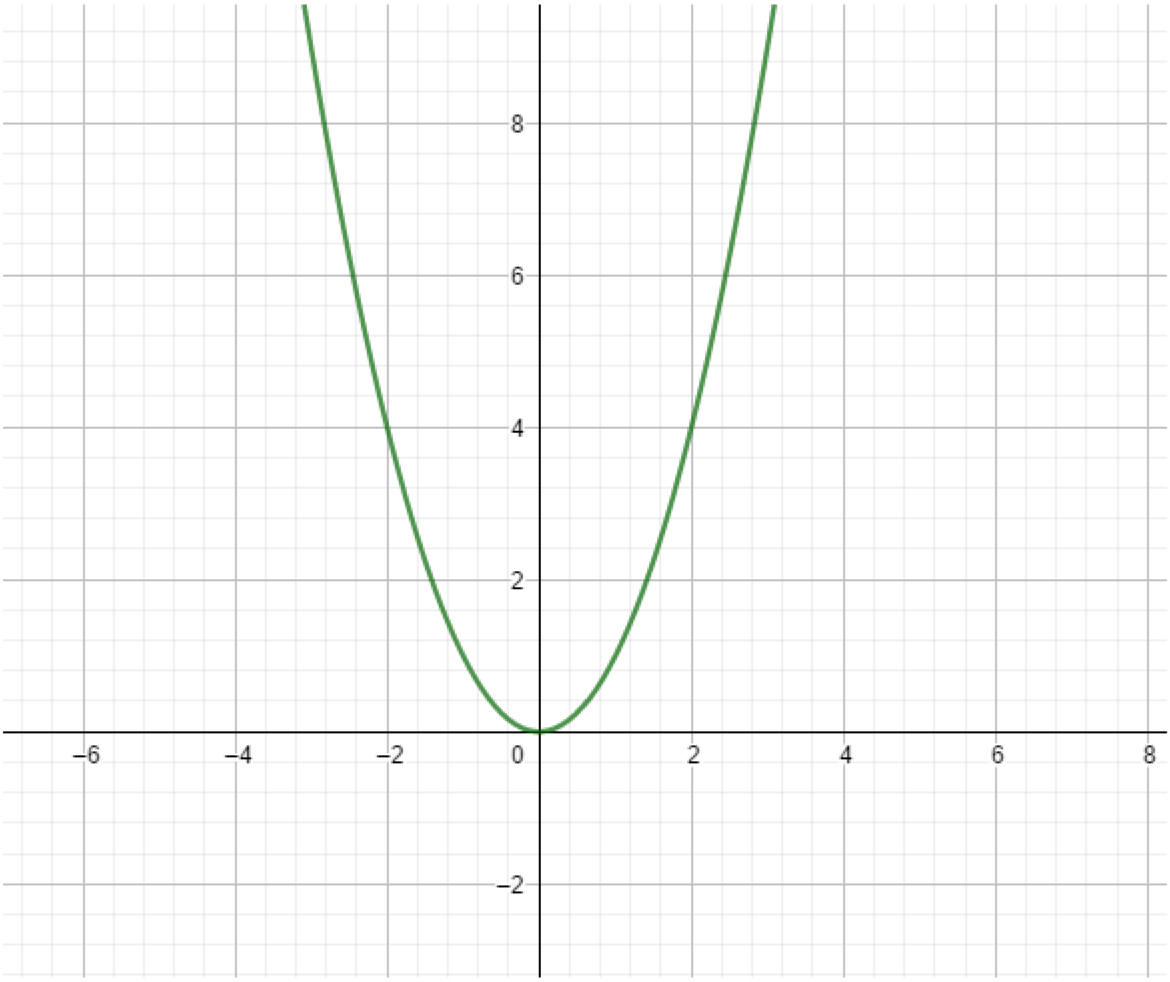



*Figure 1: A graph of x squared*

The graph above has several points that you need to know and recognise:

- Shape of the graph is a parabola
- The graph has one root which is where it touches the x axis
- The graph crosses the y axis at 0.

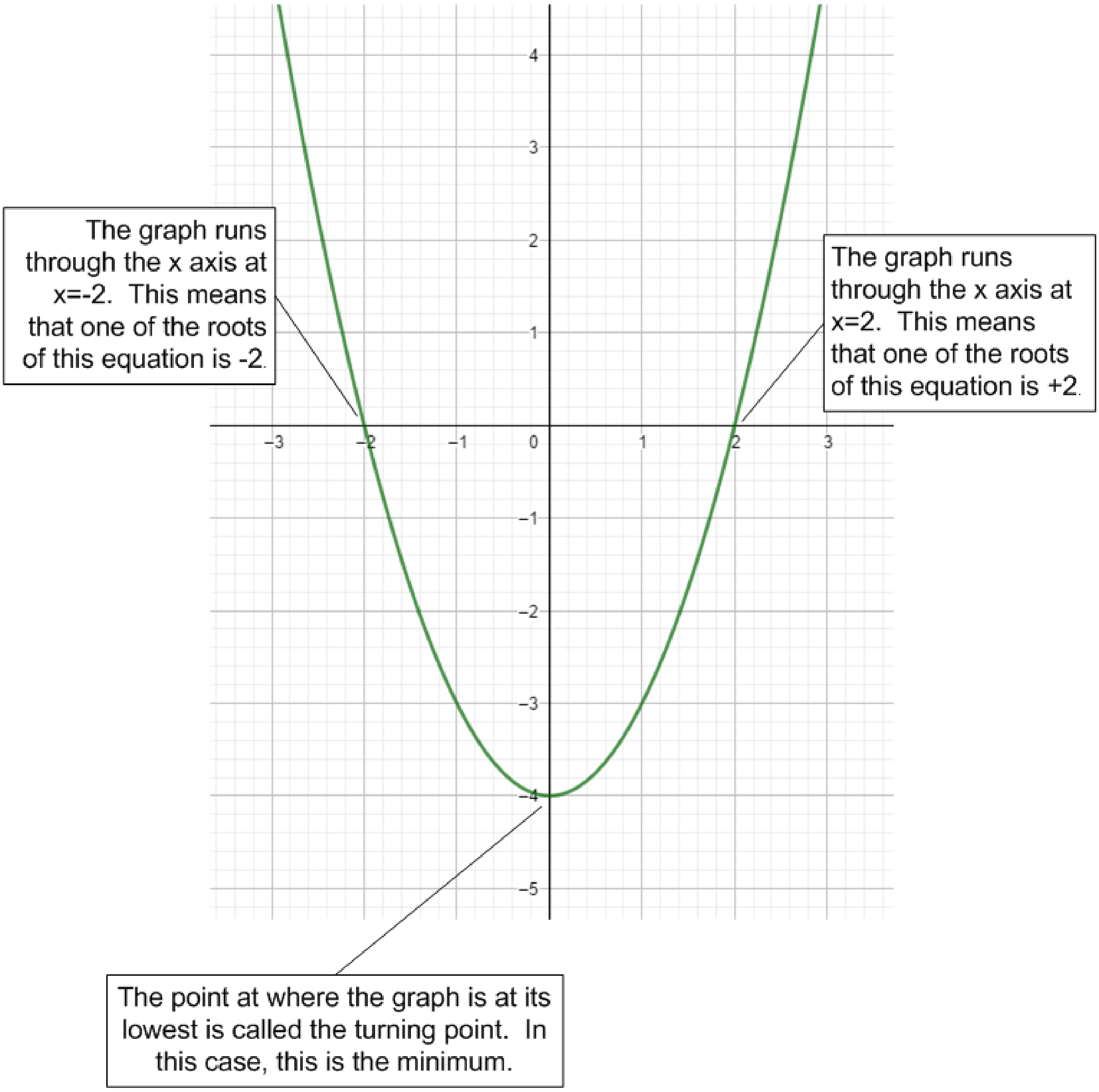



*Figure 2: x^2 -4*

- The graph is exactly the same shape as the $y=x^2$ graph shown on the previous page
- The intercept or the point through which the graph passes through the y axis is at (0,-4)
- There are two roots or solutions to this graph. These are where the graph line passes through the x-axis.
- You find the roots by factorising or using the quadratic equation.
- You find the co-ordinates of the turning point by completing the square.

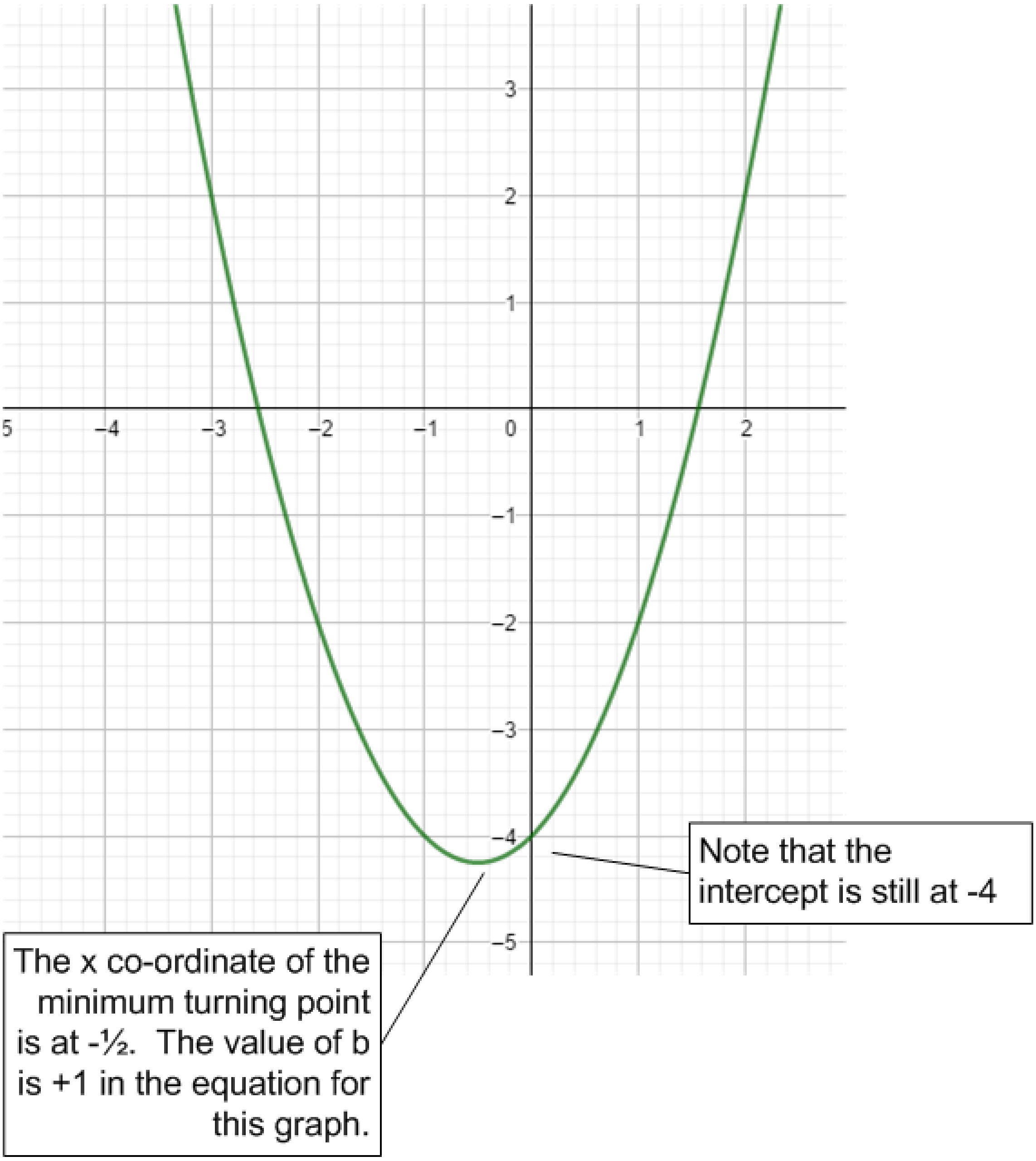



*Figure 3: x^2 + x -4*

- The shape is the same: it is just translated.
- If the equation was $ax^2 + bx - c$ then the x value of the minimum turning point is b/2
- +b moves the graph to the left. -b moves the graph to the right.

**Solving quadratic equations.**

Quadratic equations have any of two, one or no real solutions.

**What is the discriminant?**

If we have the formula $ax^2 + bx + c = 0$, then this can be solved using the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The part of the formula under the square root sign is called the ***discriminant***. That is the $b^2 - 4ac$.

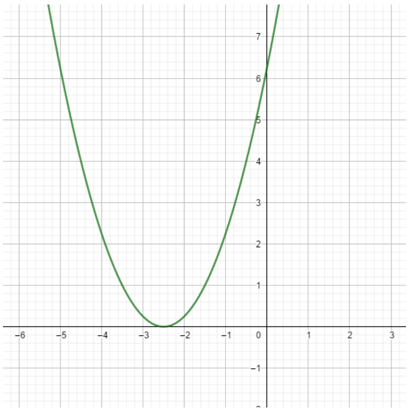
We can use the discriminant to determine whether or not there are any real solutions to a quadratic equation. This can save time in the event of their being no real solutions.

## If $b^2 - 4ac > 0$, then there are 2 real solutions to the equation.

## If $b^2 - 4ac = 0$, then there is 1 real solution to the equation.

## If $b^2 - 4ac < 0$, then there are no real solutions to the equation.

This information relates to the graphs on the following page.

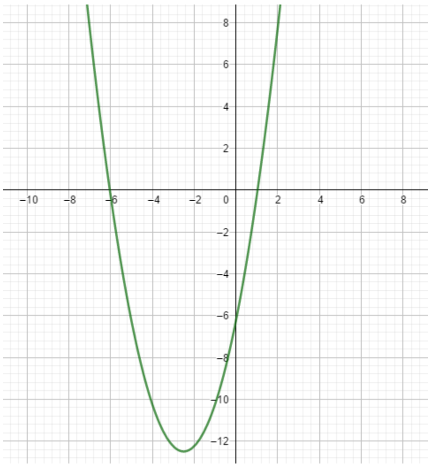


The is the graph of $y = x^2 + 5x + 6.25$.

It has one solution because the graph line touches the x axis at one point.

Examining the discriminant, we have:

$$\boldsymbol{b^2 - 4ac = 0}$$

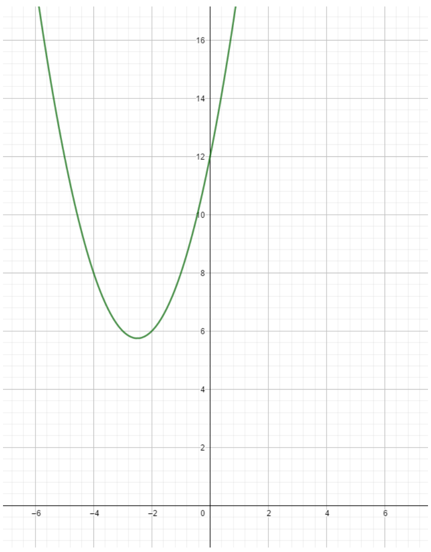


The graph of $y = x^2 + 5x - 6.25$ has two solutions because the graph crosses the x-axis twice: once on the way down and once on the way back up.

The solutions are also called the roots of the equation.

Examining the discriminant, we have:

$$\boldsymbol{b^2 - 4ac > 0}$$



The graph of $y = x^2 + 5x + 12$ has no real solutions. That is because the graph does not pass through the x axis at all.

In actual fact, the equation will have solutions using things called complex numbers, but we don't start looking at these until we are studying for a degree.

Examining the discriminant, we have:

$$\boldsymbol{b^2 - 4ac < 0}$$

**Exercise 1: Using the discriminant**

How many real solutions do the following equations have?

a) $3x^2 - 15x + 7 = 0$

b) $7x^2 + 5x - 21 = 0$

c) $x^2 - 13x + 10 = 0$

d) $4x^2 - 12x + 9 = 0$

e) $8x^2 + 10x + 12 = 0$

There are three types of calculation that you need to do with quadratic equations. Sometimes on your exam, the one you will need to use will be made explicit.

**Solve by Factorising $ax^2 + bx + c = 0$**

**Find factor pair of ac that add up to b.**

**Divide both of the factors by a and then simplify.**

**The denominator of each fraction is the x coefficient. The numerator is the value to be added to the x term in bracket.**

$$14x^2 - x - 3 = 0$$

**ac= -42 so factor pairs are {-1, 42}, {1, -42}, {-2, 21}, {-21, 2}, {-3, 14}, {-14,3}, {-6, 7}, {-7, 6}**

**b= -1 so -7 and 6 is the factor pair that works for this equation.**

**Divide both by a=14**

$$\frac{-7}{14} \ and \ \frac{6}{14}$$

**Simplify the fractions**

$$\frac{-1}{2} \ and \ \frac{3}{7}$$

**The denominator is the x co-efficient**

**(2x – 1) and (7x + 3)**

**Then solve to get x=1/2 and x=3/7**

**Solve by using the equation $3x^2 - 17x + 12 = 0$.**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-17) \pm \sqrt{(-17)^2 - (4 \times 3 \times 12)}}{2 \times 3}$$

Examining the discriminant: 289 – 144 = 145 which is positive so there will be two roots to the equation.

$$x = \frac{17 \pm \sqrt{145}}{6}$$

$$\therefore x = 0.8264009035 \ or \ x = 4.840265763$$

**Completing the square**

Completing the square is a way of finding the co-ordinates of the turning point of a quadratic curve. The information it gives you is different to solving for the roots or solutions.

A quadratic equation in the following form is in its completed square form:

$$(x+p)^2+q$$

Where p and q give the co-ordinates (p,q) of the turning point of the parabola.

To begin with, you need to memorise two identities. They are very similar, differing only by a sign.

$$x^2+2bx+c \equiv (x+b)^2-b^2+c$$

$$x^2-2bx+c \equiv (x-b)^2-b^2+c$$

Put the following into completed square form:

1) $x^2+6x+10$

2) $x^2-6x+10$

3) $x^2+8x+15$

4) $x^2+10x-7$

5) $x^2+12x+9$

6) $x^2+5x+3$

7) $x^2-9x+17$