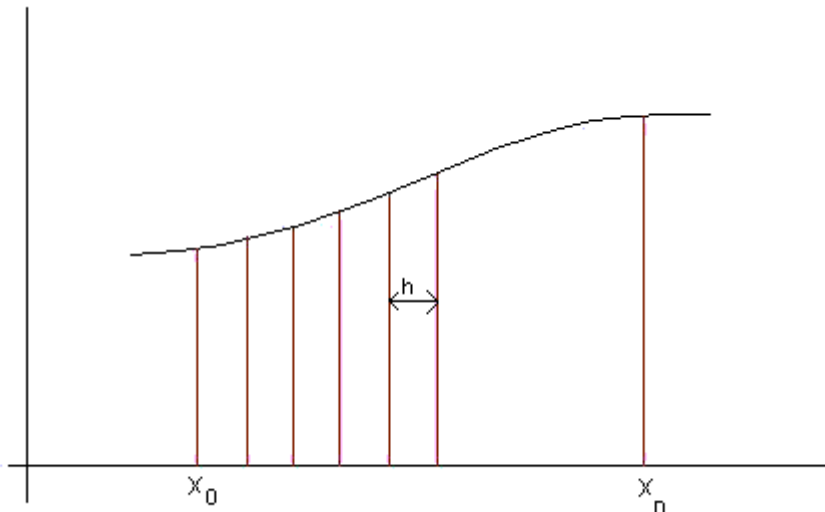


The Trapezium Rule

The trapezium rule is a way of estimating the area under a curve. We know that the area under a curve is given by integration, so the trapezium rule gives a method of estimating integrals. This is useful when we come across integrals that we don't know how to evaluate.

The trapezium rule works by splitting the area under a curve into a number of trapeziums, which we know the area of.



If we want to find the area under a curve between the points x_0 and x_n , we divide this interval up into smaller intervals, each of which has length h (see diagram above).

Then we find that:

$$\int_{x_0}^{x_n} f(x) dx = \frac{1}{2} h [(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})]$$

where $y_0 = f(x_0)$ and $y_1 = f(x_1)$ etc

If the original interval was split up into n smaller intervals, then h is given by:

$$h = (x_n - x_0)/n$$

Example

Evaluate

$$\int_0^1 \sqrt{2x+1} \, dx$$

We'll use values of x at 0, 0.25, 0.5, 0.75 and 1 (I've just chosen regularly spaced values between 0 and 1).

So "h" in this case is 0.25

We need to find the value of $\sqrt{2x+1}$ at these values of x :

x	$\sqrt{2x+1}$
0	1
0.25	1.22
0.5	1.41
0.75	1.58
1	1.73

Now apply the trapezium rule:

$$\begin{aligned} & 1/2 \times 0.25 \times [1 + 1.73 + 2(1.22 + 1.41 + 1.58)] \\ & = \underline{\underline{1.39}} \end{aligned}$$