Year 9

# Pythagoras' Theorem

## Squares, Cubes, Roots and Indices

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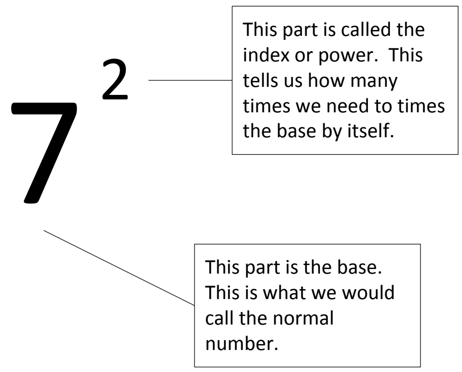
## Indices

A number has two parts: the base and the index. The base is the part we usually see. Generally, the index is 1 which doesn't change the base at all so we don't bother to write it down.

Eg




If we were reading this number aloud, we would say, "**Seven squared**," or "**Seven to the power of two.**"

The base tells us what number we are multiplying. The index, or power, tells us how many times we need to multiply that number together.

Eg

$5^3 = 5 \times 5 \times 5 = 125$ , $7^2 = 7 \times 7 = 49$

Work out the value of the following powers. Use the examples above to show you how to set it out.

1) $5^3$
2) $7^2$
3) $3^4$
4) $6^1$
5) $8^2$
6) $10^3$
7) $x^2$
8) $x^0$
9) $5^5$
10) $7^4$
11) $3^2$
12) $6^5$
13) $8^3$
14) $10^2$
15) $y^3$
16) $t^4$

Anything, be it a number, a variable, a function or a constant, to the power of zero is 1.

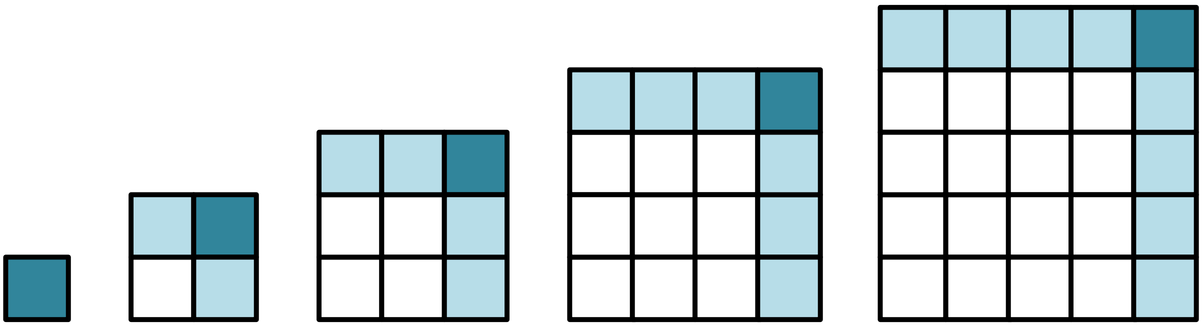
Eg: $4^0 = 1$, $8^0 = 1$, $(fred)^0 = 1$, $m^0 = 1$, $x^0 = 1$.

17) $5^0 \times 4^1 \times 3^2 \times 2^3 \times 1^4 =$
18) $6^0 \times 5^1 \times 4^2 \times 3^3 \times 2^4 =$
19) $7^0 \times 6^1 \times 5^2 \times 4^3 \times 3^4 =$
20) $8^0 \times 7^1 \times 6^2 \times 5^3 \times 4^4 =$
21) $9^0 \times 8^1 \times 7^2 \times 6^3 \times 5^4 =$
22) $5^4 \times 4^3 \times 3^2 \times 2^1 \times 1^0 =$
23) $12^0 \times 11^1 \times 10^2 \times 9^3 \times 8^4 =$

When we are looking for patterns in groups of numbers, we usually look at the difference between the numbers.

24) If you look at numbers 17 to 21 above, can you use the pattern to predict what might come next in the sequence?

Look at the square numbers between 1 and 5.



Can you use the diagram above to predict any other values of square numbers?

25) If we say that $S_n$ is the nth square number, such that if n=4, $S_4 = 16$, can you copy out and complete the following table up to and including n=12?

| | | | | | |
|---|---|---|---|---|---|
| N: | 0 | 1 | 2 | 3 | 4 |
| $S_n$: | 0 | 1 | 4 | 9 | |
| N: | 5 | 6 | 7 | 8 | 9 |
| $S_n$: | | | | | |

Using the diagram on page 1 and the information above, can you derive a formula that will predict the next square number?

Clues: You might need to include the expressions:

$$\mathbf{n^2,\ (n-1)^2 + n,\ 2n + 1,\ 5n + 1}$$

You will need to put in the previous version of n into your expression and it come up with correct number for $(n+1)^2$.

For example, if I put in 3 into the equation that I had derived, it should come up with the answer, 16 when I calculate it out. This is because:

$$(3+1)^2 = 16.$$

26) Pythagorean Triples are sets of integers that when you add the square of the lower two together, you end up with the square of the largest number.

eg $\mathbf{3}^2 + \mathbf{4}^2 = 9 + 16 = 25 = \mathbf{5}^2$

so (3, 4, 5) make a Pythagorean Triple.

See if you can find another three sets of numbers for which this works.

This formula might help you:

If you choose two numbers of opposite parity (one odd, one even) where m> n,

$2mn,\ m^2 - n^2,\ m^2 + n^2$

So if I choose m=2 and n=1:

$2mn = (2\times m\times n) = (2\times 2\times 1) = 4$

$m^2 - n^2 = (m\times m) - (n\times n) = (2\times 2) - (1\times 1) = 3$

$m^2 + n^2 = (m\times m) + (n\times n) = (2\times 2) + (1\times 1) = 5$

So choosing m=2 and n=1 leads to a (3, 4, 5) Pythagorean Triple.

Try m=4, n=1 to see if it works.

Other suggestions: m=3, n=2

m=5, n=4

m=7, n=2

NOTE: if **m** is odd, **n** has to be even and vice versa. Also, **m** needs to be greater than **n**.