



Triangle ABC is circumscribed by a circle whose centre is placed at O such that AOC form the diameter of the circle, which runs along the x-axis. AOC is subtended at B. Line AB is extended so that it runs through the points (0, -2) and (16,12). Point D is a point on the extended line AB such that CD forms the line, $x=9$. The co-ordinates on the x and y axes are placed at 1 cm intervals. What is the area of the circle outside the triangle ABC?

To find the equation of the line AB:

The line is a straight line and so has the general equation $y=mx+c$.

c forms the intercept and is therefore (-2)

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

$$= \frac{(12 - (-2))}{(16 - 0)}$$

$$m = \frac{7}{8}$$

So the equation for the line AB is

$$y = \frac{7}{8}x - 2$$

To calculate point A:

$$y = \frac{7}{8}x - 2$$

$$y + 2 = \frac{7}{8}x$$

$$\frac{8(y + 2)}{7} = x$$

$$\frac{8(0 + 2)}{7} = x$$

$$x = \frac{16}{7}$$

So the co-ordinates of point A are (16/7,0).

To calculate the diameter of the circle:

$$\begin{aligned} \text{Diameter of the Circle} &= x_C - x_A \\ &= 9 - (16/7) \\ &= 6\frac{5}{7} \\ &= \frac{47}{7} \end{aligned}$$

To calculate the Area of the circle:

For the purpose of this calculation, we shall take $\pi \approx \frac{22}{7}$.

$$\begin{aligned} A &= \pi r^2 \\ A &= \left(\frac{22}{7}\right) \left(\frac{47}{7}\right)^2 \\ &= \frac{48598}{343} \\ &= 141\frac{235}{343} \end{aligned}$$

To calculate the co-ordinates of D:

$$y = \frac{7}{8}x - 2$$

$$y = \frac{7}{8}(9) - 2$$

$$= \frac{47}{8}$$

So the co-ordinates of D are (9, 47/8).

To calculate angle CAD (**making sure your calculator is set to DEGREES**):

Let $\alpha = \text{Angle CAD}$

$$\tan \alpha = \frac{\textit{Opposite}}{\textit{Adjacent}}$$

$$\tan \alpha = \frac{47/8}{47/7}$$

$$\tan \alpha = \frac{7}{8}$$

$$\tan^{-1} \alpha = 41.18592517^\circ$$

To calculate the distance, BC:

$$\sin \alpha = \frac{\textit{Opposite}}{\textit{Hypotenuse}}$$

$$\textit{Opposite} = \textit{Hypotenuse}(\sin \alpha)$$

$$= \frac{47}{7}(\sin 41.18592517)$$

$$= 4.421388081$$

To calculate area of triangle ABC:

$$\textit{Area}_{\textit{Triangle}} = \frac{1}{2}(\textit{base} \times \textit{height})$$

$$= \frac{1}{2}\left(4.421388081 \times \frac{47}{7}\right)$$

$$= 14.84323142$$

To calculate the area of the circle outside the triangle ABC:

$$\textit{Area}_{\textit{required}} = \textit{Area}_{\textit{circle}} - \textit{Area}_{\textit{triangle}}$$

$$\begin{aligned} &= 141 \frac{235}{343} - 14.84323142 \\ &= 126.8418998 \text{ cm}^2 \end{aligned}$$